Note Title EE 362 Spring 2019 3/14/2019

$$x[n] = \begin{cases} 1, & n = -q, -q+1, \ldots, q-1, q \\ 0, & \text{o.w} \end{cases} = p_{2q}[n]$$

$$X(\Omega) = \sum_{n=-\infty}^{\infty} x[n] e^{-j\Omega n} = \sum_{n=-q}^{q} e^{-j\Omega n} \xrightarrow{k = n+q} \sum_{k=0}^{2q} \left(e^{-j\Omega}\right)^{k-q}$$

$$= \sum_{k=0}^{2q} e^{-j\Omega k} e^{j\Omega q} = e^{j\Omega q} \sum_{k=0}^{2q} e^{-j\Omega k} = e^{j\Omega q} \sum_{k=0}^{2q} \left(e^{-j\Omega}\right)^{k}$$

$$\sum_{i=0}^{n} a^i = \frac{1-a^{n+1}}{1-a} \qquad e^{-2jq\Omega - j\Omega}$$

$$\therefore X(\Omega) = e^{j\Omega q}\left(\frac{1-\left(e^{-j\Omega}\right)^{2q+1}}{1-e^{-j\Omega}}\right)$$

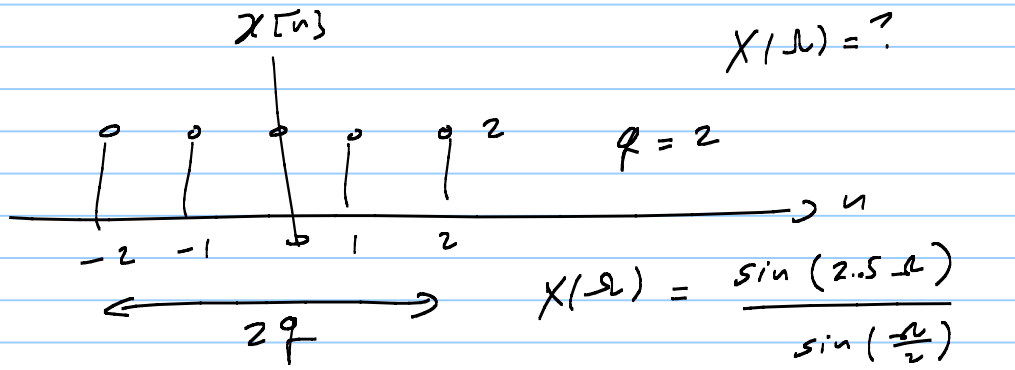
$$\cos(\theta) = \frac{e^{j\theta} + e^{-j\theta}}{2} \qquad \sin(\theta) = \frac{e^{j\theta} - e^{-j\theta}}{2j}$$

$$= \frac{e^{j\Omega q} - e^{-j(q+1)\Omega}}{1-e^{-j\Omega}} * \frac{e^{j\frac{\Omega}{2}}}{e^{j\frac{\Omega}{2}}}$$

$$= \frac{\left(e^{j(q+\frac{1}{2})\Omega} - e^{-j(q+\frac{1}{2})\Omega}\right)/2j}{\left(e^{j\frac{\Omega}{2}} - e^{-j\frac{\Omega}{2}}\right)/2j}$$

$$= \frac{\sin\left((q+\frac{1}{2})\Omega\right)}{\sin\left(\frac{\Omega}{2}\right)}$$

$$p_{2q}[n] \longleftrightarrow \frac{\sin\left((q+\frac{1}{2})\Omega\right)}{\sin\left(\frac{\Omega}{2}\right)}$$

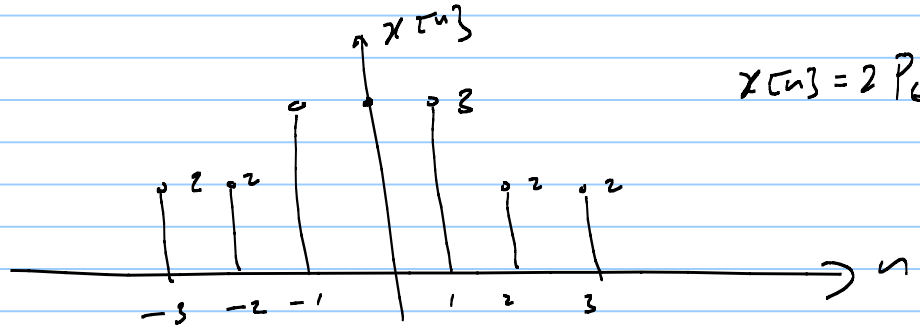


$$X(\Omega) = \frac{\sin(2.5\Omega)}{\sin\left(\frac{\Omega}{2}\right)}$$

$$x[n] = \begin{cases} 0, & n<0,\ n>P \\ a^n, & 0 \le n \le P \end{cases}, \quad P \text{ is positive integer}$$

$$X(\Omega) = \sum_{n=-\infty}^{\infty} x[n]\, e^{-j\Omega n} = \sum_{n=0}^{P} a^n e^{-j\Omega n} = \sum_{n=0}^{P} \left(a e^{-j\Omega}\right)^n$$

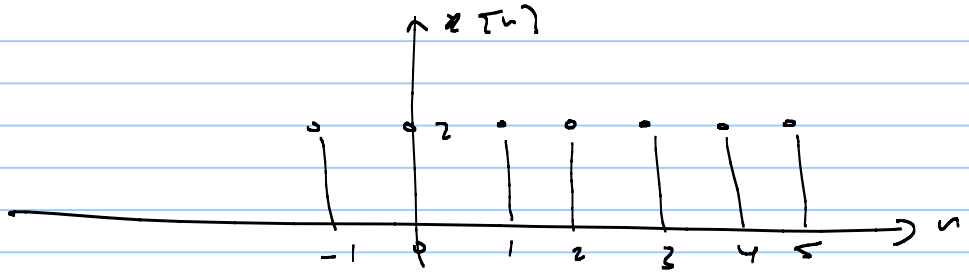
$$X(\Omega) = \frac{1-(a e^{-j\Omega})^{P+1}}{1-a e^{-j\Omega}}$$



$$x[n] = 2\,p_6[n] + p_2[n]$$

$$p_{2q}[n] \longleftrightarrow \frac{\sin\left(\left(q+\frac{1}{2}\right)\Omega\right)}{\sin\left(\frac{\Omega}{2}\right)}$$

$$X(\Omega) = 2\,\frac{\sin(3.5\Omega)}{\sin\left(\frac{\Omega}{2}\right)} + \frac{\sin(1.5\Omega)}{\sin\left(\frac{\Omega}{2}\right)}$$



$$x[n] = 2\,p_6[n-2]$$

$$2\,p_6[n] \longleftrightarrow 2\,\frac{\sin(3.5\Omega)}{\sin(\Omega/2)} \Rightarrow X(\Omega) = 2\,\frac{\sin(3.5\Omega)}{\sin\left(\frac{\Omega}{2}\right)}\, e^{-j2\Omega}$$

If

$$X(\Omega) = 2\,\frac{\sin(3.5\Omega)}{\sin(\frac{\Omega}{2})}$$

If $v[n] = x[-n]$; find $V(\Omega) = X(\Omega)^* = 2\,\frac{\sin(3.5\Omega)}{\sin(\frac{\Omega}{2})}$

If $X(\Omega) = \frac{1}{1 - 2e^{-j3\Omega}}$ and $v[n] = x[-n]$

find $V(\Omega)$

$$V(\Omega) = \frac{1}{1 - 2e^{j3\Omega}}$$

If $v[n] = n\,x[n]$; find $V(\Omega)$

$$V(\Omega) = j\,\frac{d}{d\Omega}\left(\frac{1}{1 - 2e^{-j3\Omega}}\right)$$

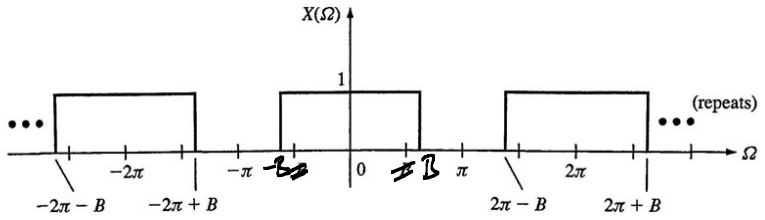
$$= j\,\frac{(1 - 2e^{-j3\Omega})(0) - (1)\left(0 - (-j3)\,2e^{-j3\Omega}\right)}{(1 - 2e^{-j3\Omega})^2}$$

$$= \frac{-3\,2e^{-j3\Omega}}{(1 - 2e^{-j3\Omega})^2}$$

If $v[n] = x[n]\,e^{-j2n} \Rightarrow V(\Omega) = 2\,\frac{\sin(3.5(\Omega + 2))}{\sin\left(\frac{\Omega + 2}{2}\right)}$

If $v[n] = x[n]\sin(10n)$

$$\Rightarrow V(\Omega) = \frac{j}{2}\left[\frac{2\sin(3.5(\Omega + 10))}{\sin\left(\frac{\Omega + 10}{2}\right)} - \frac{2\sin(3.5(\Omega - 10))}{\sin\left(\frac{\Omega - 10}{2}\right)}\right]$$



$$P_{2B}(t) \longleftrightarrow 2B\,\mathrm{sinc}\left(\frac{2B\omega}{2\pi}\right)$$

$$2B\,\mathrm{sinc}\left(\frac{2Bt}{2\pi}\right) \longleftrightarrow 2\pi P_{2B}(\omega)$$

$$\frac{B}{\pi}\,\mathrm{sinc}\left(\frac{Bt}{\pi}\right) \longleftrightarrow P_{2B}(\omega)$$